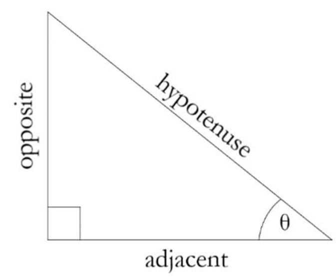
Trigonometric Identity II

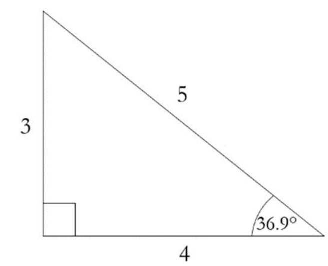
$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

This is a helpful equation used to relate the functions sine (otherwise known as sin) and cosine (otherwise known as cos) and tangent (otherwise known as tan). The ≡ symbol means "identical to" (i.e. tan theta is identical to sin theta over cos theta). This symbols means the relationship is always true, regardless of the value of θ. θ is a placeholder for an angle, and for this identity to work the angle must be the same for sine, cosine and tan.



We know, from SOH CAH TOA, that for a triangle $\sin\theta = \frac{opposite}{hypotenus}$, $\cos\theta = \frac{adjacent}{hypotenuse}$ and $\tan\theta = \frac{opposite}{adjacent}$

Therefore, using the example to the right, $\sin 36.9° = \frac{3}{5}$, $\cos 36.9° = \frac{4}{5}$ and $\tan 36.9° = \frac{3}{4}$



N.B. 36.9° is a rounded value, the real value is 36.869897…°

$$\frac{3}{5} \div \frac{4}{5} = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$$

See above how dividing one fraction by another is the same as multiplying that fraction by the reciprocal of the other. A reciprocal is a quantity that results from a number being divided into one e.g. $1 \div \frac{4}{5} = \frac{5}{4}$ and $1 \div 5 = \frac{1}{5}$, therefore the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$ and the reciprocal of 5 is $\frac{1}{5}$.

Proof

Using SOH CAH TOA

$$\frac{\sin\theta}{\cos\theta} = \frac{opposite}{hypotenuse} \div \frac{adjacent}{hypotenuse}$$

Using the reciprocal rule from above

$$= \frac{opposite}{\cancel{hypotenuse}} \times \frac{\cancel{hypotenuse}}{adjacent}$$

The value for the length of the hypotenuse cancels, leaving 1

$$= \frac{opposite}{1} \times \frac{1}{adjacent}$$

Multiplying these fractions gives

$$= \frac{opposite}{adjacent}$$

Again, using SOH CAH TOA, we find that

$$= \tan\theta$$

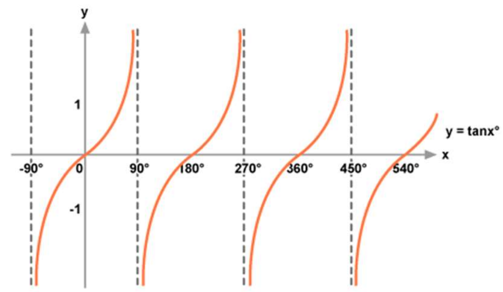
Therefore, we know that

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

Points to consider

Looking at the graph of $y = \tan x$ we will see that there are asymptotes at 90°, 270°, 450° and so on. This means that the values of $\tan 90°, \tan 270°, \tan 450°$ etc. are undefined.

The reason for this is because when x is any of these values, $\cos x = 0$ (i.e. $\cos 90° = 0, \cos 270° = 0, \cos 450° = 0$). By the equation $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$, which can otherwise be written as $\tan x \equiv \frac{\sin}{\cos x}$, for these angles $\sin x$ would be divided by 0. It is not possible for any value to be divided by 0 and so $\tan x$ for these angles is not defined.



See also
- Sine, Cosine and Tangent (SOH CAH TOA)
- Trigonometric Identity I

References

Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. p.209